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Supergravity description of the large-N noncommutative dipole field theories

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Abstract: We consider system of Dp-branes in the presence of a nonzero B field with one leg along brane worldvolume and the other transverse to it. We study the corresponding supergravity solutions and show that the worldvolume theories decouple from gravity for $p \leq 5$. Therefore these solutions provide dual description of large N noncommutative dipole field theories. We shall only consider those systems which preserve 8 supercharges in the branes worldvolume. We analyze the system of M5-branes and NS5-branes in the presence of nonzero C field and RR field with one leg along the transverse direction and the others along the worldvolume of the brane, respectively. This could provide a new deformation of $(2,0)$ and little string field theories. Finally, we study the Wilson loops using the dual gravity descriptions.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

It was believed for a long time that the four dimensional large N gauge theory have a string theory description [1], though the strings live in more than four dimensions [2]. The well-known example of this correspondence is $\mathcal{N}=4$ SU(N) SYM theory in four dimensions whose string theory dual is ten dimensional superstring theory known as type-IIB [3, 4, 5]. By now we know a large class of examples in the context of AdS/CFT correspondence (see [6] for a review) which relates field theories without gravity to superstring (string) theories on certain curved background. This correspondence naturally arises when considering Dp-branes in a limit where the worldvolume field theory decouples from the bulk gravity.

This correspondence has been generalized for the cases where the field theory is non-local, e.g. for non-commutative field theories. From string theory side, these theories can be obtained on the D-brane worldvolume in the presence of a non-zero B field [7]. With N coinciding Dp-branes in the presence of a non-zero B field the worldvolume theory is deformed to a U(N) noncommutative SYM theory [8].

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As in the case with $B = 0$, there exists a limit where the bulk gravity decouples from the worldvolume noncommutative field theory [8, 9] and a correspondence between string theory on curved background with B field and noncommutative field theory is expected. Actually this issue has been investigated during last three years in several papers including [10]-[17].

Turning on a B field on the D-brane worldvolume can be viewed, via AdS/CFT correspondence, as a perturbation of the worldvolume field theory by an operator of dimension 6. For example in the D3-brane case, from the four dimensional superconformal Yang-Mills theory point of view the bosonic part of the dimension 6 operator is given by [18, 19]

$$\mathcal{O}_{\mu\nu} = \frac{1}{2g_{\rm YM}^2}\,{\rm Tr}\left(F_{\mu\delta}F^{\delta\rho}F_{\rho\nu} - F_{\mu\nu}F^{\rho\delta}F_{\rho\delta} + 2F_{\mu\rho}\sum_{i=1}^{6}\partial_\nu\phi^i\partial^\rho\phi^i - \frac{1}{2}F_{\mu\nu}\sum_{i=1}^{6}\partial_\rho\phi^i\partial^\rho\phi^i\right), \tag{1.1}$$

where $g_{\rm YM}$ is the SYM coupling, $F_{\mu\nu}$ is the $\mathrm{U}(N)$ field strength and ϕ^i, $i = 1, \ldots, 6$ are the adjoint scalars. This deformed theory by the operator $\mathcal{O}_{\mu\nu}$ can be extended to a complete theory with a simple description which is noncommutative SYM theory.

On the other hand we could deform the theory by an other operator. In particular one can consider an operator with dimension 5 whose bosonic part is given by [20]

$$\mathcal{O}_\mu^{ij} = \frac{i}{g_{\rm YM}^2}\,{\rm Tr}\left(F_{\mu\nu}\phi^{[i}D^\nu\phi^{j]} + \sum_{k=1}^{6}D_\mu\phi^{[k}\phi^i\phi^{j]}\right), \tag{1.2}$$

where D_μ is the covariant derivative with respect to gauge field A_μ of field strength $F_{\mu\nu}$ and $[\cdots]$ is complete antisymmertization.

The deformed theory by this vector operator can be considered as low energy limit of "*dipole theory*" which is a non-local field theory such that some fields of the theory have dipoles with constant length. Such a theory was discussed in [20] in the context of T-duality in noncommutative geometry. In fact the T-duality of the gauge theory on a noncommutative torus can be extended to include fields with twist boundary condition.

From string theory point of view this non-local theory can be obtained from worldvolume theory of a D-brane in the presence of non-zero B field with one leg along the brane worldvolume and the other along the transverse directions to the brane. This brane configuration was studied in [20, 21, 22, 23] where the twisted compactification was introduced. This twisted compactification leads us to introduce a new type of star product between the fields at the level of effective field theory. This is the aim of this article to study the dipole field theory using gravity via AdS/CFT correspondence.

The plan of this paper is a following: In section 2 we will review the dipole field theory. In section 3 we shall construct the supergravity solutions corresponding to the D-branes in the presence of non-zero B field with one leg along the brane worldvolume and the other transverse to it. We will show that for this Dp-brane system the worldvolume theory decouples from the gravity for $p \leq 5$. In section 4 we will use these supergravity solutions to study dipole field theories via AdS/CFT correspondence. In section 5 we shall consider the type-II NS5-branes in the presence of RR field with one leg along the transverse directions of the branes and the others along the NS5-branes worldvolume. We then proceed to study

M5-brane in the presence of C field with two legs along the M5-brane worldvolume and the other transverse to the brane. These solutions could provide new deformations of little string theory and $(2,0)$ theory. In section 6 we shall compute the Wilson loops in the dipole gauge theory using the dual gravity description. The section 7 is devoted to conclusions and comments.

2. Dipole field theory

Dipole field theory can be thought as a generalization of ordinary field theory on a commutative space which is non-local theory and breaks Lorantz invariance. In order to turn the ordinary theory to non-local dipole theory one assigns, to every fields Φ_a, a constant dipole vector with dipole length L_a and define the dipole star product as

$$\Phi_a * \Phi_b(x) \equiv \Phi_a\left(x - \frac{L_b}{2}\right)\Phi_b\left(x + \frac{L_a}{2}\right). \tag{2.1}$$

This product is associative if the vector assignment is additive, that is, $\Phi_a * \Phi_b$ has dipole vector $L_a + L_b$. Moreover, demanding $\Phi_a^\dagger * \Phi_a$ to be real fixes the dipole length of $\Phi_a^\dagger$ to be minus of that of Φ_a. As a result, the gauge field has zero dipole length.

In order to write an action for a dipole theory it is enough to replace the product of fields in the commutative action with the dipole star product. We note, however, that one should insert the proper dipole length for all fields. In particular, any term in the proposed action should have total zero dipole length [25].

Since the gauge field has zero dipole length we cannot have nontrivial pure supersymmetric Yang-Mills theory. At least one needs to have a hyper-multiplet. In this case while the vector-multiplet has zero dipole length, the hyper-multiplet could have a non-zero dipole. Therefore the maximal possible supersymmetric noncommutative dipole theories, in dimensions less than seven, have 8 supercharges. In 6-dimensions it is $\mathcal{N} = 1$ SYM theory coupled to a hyper-multiplet and in 4-dimensions it is $\mathcal{N} = 2$ coupled to an adjoint hyper-multiplet, and etc. In fact, these are the theories one can get from string theory.

In the next section we present the supergravity solution of D-brane in type-II string theories in the presence of a non-zero B field with one leg along the worldvolume. These solutions would provide a string theory realization of dipole theories. Although in general such a configuration will break the supersymmetry completely, we shall consider the cases which have maximal supersymmetry that are theories with 8 supercharges.

3. The Supergravity solution

In order to find supergravity solutions corresponding to the dipole field theories in various dimensions we start from type-II string theories on a space that is $R^{9,1}$ modded out by the isometry

$$\mathcal{U} : (x_0, \ldots, x_p, \{x_a\}_{a=p+1}^{9}) \mapsto \left(x_0, \ldots, x_p + 2\pi\beta_p, \left\{\sum_{b=p+1}^{9} O_{ab}x_b\right\}_{a=p+1}^{9}\right). \tag{3.1}$$

Here $O \in \mathrm{SO}(9-p)$ is an orthogonal matrix. Actually this is the generalization of $p=3$ case considered in [23]. The explicit from of O is given by $O = e^{2\pi i \beta_p M/\alpha'}$ where M is a finite matrix of the Lie algebra $\mathrm{so}(9-p)$ with dimension of length.

Now we want to probe these backgrounds with a system of N D$(p-1)$-branes with worldvolume directions $(x_0, \ldots, x_{p-1})$ and taking $\beta_p \to 0$ limit. When $M=0$ we can perform T-duality finding normal Dp-brane. On the other hand for the case of $M \neq 0$ performing T-duality will end up with a Dp-brane whose low energy effective theory is going to be a dipole theory [23].

To find the explicit supergravity solution of Dp-brane describing the dipole theory via the Maldacena conjecture [3], we start with the following type-II supergravity solution describing N coincident extremal for $D_p$brane (in string frame)[26]

$$\begin{aligned} ds^2 &= f^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + f^{1/2}(dx_{p+1}^2 + \cdots + dx_9^2)\,, \\ e^{2\Phi} &= g_s^2 f^{\frac{3-p}{2}}\,, \qquad f = 1 + \frac{(2\pi)^{p-2} c_p N g_s l_s^{7-p}}{r^{7-p}}\,, \qquad C_{0,\ldots,p} = -f^{-1}\,, \end{aligned} \tag{3.2}$$

where $c_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma(\frac{7-p}{2})$.

The supergravity solution of a D$(p-1)$-brane smeared in x_p direction can be obtained by wrapping the above supergravity solution on a circle along x_p with radius β_p and then T-dualizing in this direction. Using T-duality transformation [27], one finds

$$\begin{aligned} ds^2 &= f^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_{p-1}^2) + f^{1/2}\left(\frac{\alpha'^2}{\beta_p^2}d\hat{x}_p^2 + dx_{p+1}^2 + \cdots + dx_9^2\right), \\ e^{2\phi} &= \frac{\alpha'}{\beta_p^2} g_s^2 f^{\frac{4-p}{2}}\,, \end{aligned} \tag{3.3}$$

where $\hat{x}_p$ is now dimensionless angular coordinate with period $\hat{x}_p \sim \hat{x}_p + 2\pi$.

By making use of the modded out isometry (3.1) we can add a twist to the transverse directions $x_{p+1}, \ldots, x_9$ as we go around the circle x_p. Under the action of O in (3.1) we have

$$\delta x_a = \sum_{b=p+1}^{9} \Omega_{ab} x_b d\hat{x}_p\,, \qquad a = p+1, \ldots, 9\,, \tag{3.4}$$

where Ω_{ab} is an element of the Lie algebra $\mathrm{so}(9-p)$. As a result the metric (3.3) changes to

$$ds^2 = f^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_{p-1}^2) + f^{1/2}\left(\frac{\alpha'^2}{\beta_p^2}d\hat{x}_p^2 + \sum_{a=p+1}^{9}\left(dx_a - \sum_{b=p+1}^{9}\Omega_{ab}x_b d\hat{x}_p\right)^2\right). \tag{3.5}$$

Expanding this out, we get

$$\begin{aligned} ds^2 &= f^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_{p-1}^2) \\ &\quad + f^{1/2}\left(\left(\frac{\alpha'^2}{\beta_p^2} + X^T\Omega^T\Omega X\right)d\hat{x}_p^2 + dX^T dX - 2(dX^T\Omega X)d\hat{x}_p\right), \end{aligned} \tag{3.6}$$

where X is a vector formed by the transverse directions, i.e. $X^T = (x_{p+1}, \ldots, x_9)$.

Finally, once again, we can apply T-duality on $\hat{x}_p$ direction. Doing so, in the limit of $\beta_p \to \infty$ keeping $\beta_p \Omega = M$ fixed and setting $x_p = \beta_p \hat{x}_p$, one finds

$$\begin{aligned} ds^2 &= f^{-1/2}\left(-dt^2 + dx_1^2 + \cdots + dx_{p-1}^2 + \frac{\alpha'^2 dx_p^2}{\alpha'^2 + r^2 n^T M^T M n}\right) + \\ &\quad + f^{1/2}\left(dr^2 + r^2 dn^T dn - \frac{r^4 (n^T M^T dn)^2}{\alpha'^2 + r^2 n^T M^T M n}\right), \\ e^{2\phi} &= \frac{\alpha'^2 g_s^2 f^{\frac{3-p}{2}}}{\alpha'^2 + r^2 n^T M^T M n}, \\ \sum_{a=p+1}^{9} B_{pa} dx_a &= -\frac{r^2 dn^T M n}{\alpha'^2 + r^2 n^T M^T M n}, \end{aligned} \tag{3.7}$$

where n is unit vector defined by $X = rn$ with $|n|^2 = 1$.

In general this background breaks the supersymmetry completely, though depending on the rank of matrix M some supersymmetries are left. We will return to this point in the next section. Another problem one has to be considered is the stability of the solution. It is not obvious if the solutions we found are stable. Nevertheless taking matrix M such that the solutions preserve some amount of supersymmetries, as we will do in the next section, would hopefully lead to stable solutions.

On the other hand, given a general supergravity solution of a system of branes, it is not clear whether the solution would give a well-defined description of some field theory. In fact, we must check and see whether there is a well-defined field theory on the brane worldvolume which decouples from bulk gravity. To see this, one might calculate scattering amplitude for gravitons [28, 29]. To do this, we can compute the gravitons absorption cross section. If there is a limit (decoupling limit) where the gravitons absorption cross section vanishes, we have a field theory which decouples from the gravity. Alternatively, one can evaluate the potential that the gravitons feel because of the brane. Having a decoupled theory can be seen from the shape of the potential in the decoupling limit. Actually, for those branes which their worldvolume decouple from gravity, the potential develops an infinite barrier separating the space into two parts: bulk and brane. In this case the bulk's modes can not reach the brane because of this infinite barrier, and the same for brane's modes. Therefore the theory decouples from the bulk. As an example, for D6-brane there is no such a barrier and therefore we expect not to have a 7-dimensional gauge theory living on its worldvolume. This is also the case even with a non-zero B field [30]. In following we are going to compute the potential that the gravitons see because of the supergravity solution (3.7).

Let us perturb the metric of the background (3.7) by

$$g_{ij} = \bar{g}_{ij} + h_{ij} \qquad i, j = 0, \ldots, 9, \tag{3.8}$$

where by $\bar{g}_{ij}$ we denote the background metric (3.7) and h_{ij} is the perturbation. We consider s-wave gravitons with momenta along the brane

$$h_{ij} = \epsilon_{ij} h(r) e^{ik_\mu x^\mu}, \qquad \mu = 0, \ldots, p. \tag{3.9}$$

We choose the gauge $h_{i\mu}k^{\mu} = 0$ that keeps transversal gravitons. We will also choose the garvitons with polarization along the brane, i.e. $\epsilon_{ab} = 0$ for $a, b = p+1, \ldots, 9$. Let $k_\mu = \omega\delta_{0,\mu}$. The other possibilities do not change the physical results as far as the decoupling is concerned. Using the linearized equations of motion of type-II supergravity we find following equation for transverse gravitons

$$\partial_i \left(\sqrt{-g}\, e^{-2\phi} g^{ij} \partial_j \Phi \right) = 0 \tag{3.10}$$

with $\Phi = h(r)e^{ik_\mu x^\mu}$. From this equation one can read the potential by writing it in the form of a Schrödinger-like equation. In general it might be difficult to do this for most general form of supergravity solution (3.7). But we have done this for a particular form of matrix M corresponding to the cases that preserve eight supercharges (see next section). In all cases the equation can be simplified and we get the following Schrödinger-like equation

$$\partial_\rho^2 \psi(\rho) + V_p(\rho)\psi(\rho) = 0\,, \tag{3.11}$$

where

$$V_p(\rho) = -\left(1 + \frac{c_p N g_s (\omega l_s)^{7-p}}{\rho^{7-p}} \right) + \frac{(8-p)(6-p)}{4\rho^2} \tag{3.12}$$

with $\rho = \omega r$.

This potential is the same as once found in [30] for the ordinary D-branes as well as branes in the presence of B field. Therefore we conclude that we have a decoupled dipole theory living on the worldvolume of Dp-brane for $p \leq 5$.

In next section we shall study the dipole field theory living on the worldvolume of D-branes in the presence of B field with one leg along the worldvolume and the other along the transverse directions to the brane using corresponding supergravity solution (3.7). We will only consider those configurations that preserve 8 supercharges. Because of the supersymmetry we expect that the solution to be stable.

4. Supergravity description of dipole gauge theory

In the previous section we have shown that the worldvolume theory on a system of Dp-brane in the presence of non-zero B field with one leg along the brane worldvolume decouples from the gravity for $p \leq 5$. Therefore this can provide a dual gravity description for dipole field theory [23] via AdS/CFT correspondence.

The decoupling limit is defined as a limit in which $\alpha' \to 0$ and keeping the following quantities fixed

$$u = \frac{r}{l_s^2}\,, \qquad \bar{g}_s = g_s l_s^{p-3}\,. \tag{4.1}$$

In this limit the supergravity solution (3.7) reads

$$\begin{aligned} l_s^{-2} ds^2 = & \left(\frac{u}{R}\right)^{\frac{(7-p)}{2}} \left(-dt^2 + dx_1^2 + \cdots + \frac{dx_p^2}{1 + u^2 n^T M^T M n} \right) + \\ & + \left(\frac{R}{u}\right)^{\frac{7-p}{2}} \left(du^2 + u^2 dn^T dn - \frac{u^4 (n^T M^T dn)^2}{1 + u^2 n^T M^T M n} \right), \end{aligned}$$

$$e^{2\phi} = \bar{g}_s^2 \frac{(R/u)^{(7-p)(3-p)/2}}{1+u^2 n^T M^T M n} ,$$
$$\sum_{a=p+1}^{9} B_{pa} dn_a = -\frac{u^2 dn^T M n}{1+u^2 n^T M^T M n} , \tag{4.2}$$

with
$$(R)^{7-p} = 2^{7-2p} \pi^{(9-3p)/2} \Gamma\left(\frac{7-p}{2}\right) g_{YM}^2 N , \qquad g_{YM}^2 = (2\pi)^{p-2} \bar{g}_s . \tag{4.3}$$

The effective dimensionless coupling constant in the corresponding dipole field theory can be defined as following [24]
$$g_{\text{eff}}^2 \sim g_{\text{YM}}^2 N u^{p-3} . \tag{4.4}$$

For the cases we will be considering the scalar curvature of the metric in eq. (4.2) has the behavior
$$l_s^2 \mathcal{R} \sim \frac{1}{g_{\text{eff}}} . \tag{4.5}$$

Thus the perturbative calculation in dipole field theory can be trusted when $g_{\text{eff}} \ll 1$, while when $g_{\text{eff}} \gg 1$ the supergravity description is valid. We note also that the expression for dilaton in (4.2) can be recast to

$$e^{\phi} = \frac{1}{N} \frac{g_{\text{eff}}^{(7-p)/4}}{(1+u^2 n^T M^T M n)^{1/2}} . \tag{4.6}$$

Keeping g_{eff} and $u^2 n^T M^T M n$ fixed we see from (4.6) that $e^{\phi} \sim 1/N$. Therefore the string loop expansion corresponds to $1/N$ expansion of dipole gauge theory. Note also that in the supergravity description of dipole field theory $u^2 n^T M^T M n$ plays the role of dipole deformation. At IR limit where $u^2 n^T M^T M n \ll 1$ the dipole effects are small and the effective description of the worldvolume theory is in terms of a ordinary field theory. In this regime the supergravity solutions (4.2) reduce to the low energy background considered in [24].

When the rank of M is less than maximal, which of course the case we are interested in, the quadratic form $n^T M^T M n$ has a locus of zeroes [23]. Form the supergravity solution (4.2) we see that locally on the zero locus, solution is the same as ordinary Dp-brane background [24].

The case of $p = 3$ has been studied in [25, 23]. Here, we will first review the D3-brane case and then we shall study the other branes.

4.1 D3-brane

The theory on the worldvolume of the N coincident ordinary D3-brane is $\mathcal{N} = 4$ SU(N) SYM theory. The $\mathcal{N} = 4$ SYM theory in four dimensions has 6 real scalars in the representation **6** of R-symmetry group SU(4) and 4 Weyl fermions in the representation **4** of SU(4). To construct the dipole theory we use the R-symmetry charges to determine the dipole vectors of the various fields. For simplicity we assume that all dipoles are along 3^{rd} direction. The dipole moment can be given by vector $V^3 \in$ su(4). Denoting the matrix representation of V^3 by U for representation **4** and by M for representation **6**, the dipole vectors of fermions and scalars are given by eigenvalues of U and M, respectively [23].

In general the number of supersymmetries that are preserved by dipole theory is determined by the rank of U. It is completely broken for U of rank 4. For U with rank three, it has one zero eigenvalue and the theory has $\mathcal{N}=1$ supersymmetry. For U of rank 2 there are two zero eigenvalues and the theory has $\mathcal{N}=2$ supersymmetry.

Being a matrix in the representation **4** of su(4) the eigenvalues of U can be given by $\alpha_1, \alpha_2, \alpha_3$ and $-\alpha_1-\alpha_2-\alpha_3$. Therefore the most general form of matrix M can be cast to the following form

$$M = \begin{pmatrix} 0 & \alpha_1+\alpha_2 & 0 & 0 & 0 & 0 \\ -\alpha_1-\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1+\alpha_3 & 0 & 0 \\ 0 & 0 & -\alpha_1-\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_2+\alpha_3 \\ 0 & 0 & 0 & 0 & -\alpha_2-\alpha_3 & 0 \end{pmatrix}. \tag{4.7}$$

This form of matrix M breaks all supersymmetries. On the other hand for $\alpha_1=0$ we left with 4 supercharges. For $\alpha_1=\alpha_2=0$ we find a configuration with 8 supercharges. Setting $\alpha_3=L$, the matrix M reads

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & -L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L \\ 0 & 0 & 0 & 0 & -L & 0 \end{pmatrix}. \tag{4.8}$$

According to the definition of M [23], we have four real scalars with dipole lengths $\pm 2\pi L$ and two Weyl fermions with dipole lengths $\pm\pi L$. These are content of an $\mathcal{N}=2$ hypermultiplet. The fields in the $\mathcal{N}=2$ vector-multiplet have no dipole and this is the reason why we have 8 supercharges.

Plugging the above form of M into (4.2), one finds

$$\begin{aligned} l_s^{-2} ds^2 &= \left(\frac{u}{R}\right)^2 \left(-dt^2 + dx_1^2 + dx_2^2 + \frac{dx_3^2}{1+u^2L^2\sin^2\theta_1\sin^2\theta_2}\right) \\ &\quad + \left(\frac{R}{u}\right)^2 \left(du^2 + u^2 d\Omega_5^2 - u^4L^2\sin^4\theta_1\sin^4\theta_2 \frac{(a_3d\theta_3 + a_4d\theta_4 + a_5d\theta_5)^2}{1+u^2L^2\sin^2\theta_1\sin^2\theta_2}\right), \\ e^{2\phi} &= \frac{\bar{g}_s^2}{1+u^2L^2\sin^2\theta_1\sin^2\theta_2}, \\ \sum_{a=3}^{5} B_{3\theta_a} d\theta_a &= -\frac{u^2L\sin^2\theta_1\sin^2\theta_2}{1+u^2L^2\sin^2\theta_1\sin^2\theta_2}(a_3d\theta_3 + a_4d\theta_4 + a_5d\theta_5), \end{aligned} \tag{4.9}$$

where $\theta_1, \dots, \theta_5$ are the angular coordinates parameterizing sphere S^5 transverse to the D3-brane, and

$$a_3 = \cos\theta_4, \qquad a_4 = -\sin\theta_3\cos\theta_3\sin\theta_4, \qquad a_5 = \sin^2\theta_3\sin^2\theta_4. \tag{4.10}$$

The locus of zeros is given by equation $\sin\theta_1 \sin\theta_2 = 0$ where the gravity solution is the same as the case without B field. The phase structure of this theory has been studied in [25].

4.2 D5-brane

As we showed in the previous section, since D6-brane does not decouple from the gravity, the next example we are going to consider is D5-brane. The low energy effective theory on the worldvolume of the ordinary D5-brane is 6-dimensional gauge theory with 16 supercharges. It has SO(4) R-symmetry. One can use R-symmetry charges to make a dipole theory out of the ordinary one. In order to have a 6-dimensional supersymmetric dipole theory we need to separate the $\mathcal{N}=2$ vector-multiplet to an $\mathcal{N}=1$ vector-multiplet plus a hyper-multiplet. Now we can assign to the fields in the hyper-multiplet a dipole vector and therefore we get $\mathcal{N}=1$ 6-dimensional supersymmetric dipole gauge theory. The dipole vectors of the scalars in the hyper-multiplet can be obtained from the eigenvalues of matrix M which can be cast to the following form

$$M = \begin{pmatrix} 0 & L & 0 & 0 \\ -L & 0 & 0 & 0 \\ 0 & 0 & 0 & L \\ 0 & 0 & -L & 0 \end{pmatrix} . \tag{4.11}$$

Plugging the above matrix in (4.2) we get

$$\begin{aligned} l_s^{-2} ds^2 &= \frac{u}{R}\left(-dt^2 + dx_1^2 + \cdots + \frac{dx_5^2}{1+u^2L^2}\right) + \\ &\quad + \frac{R}{u}\left(du^2 + u^2 d\Omega_3^2 - \frac{u^4L^2}{1+u^2L^2}(a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3)^2\right), \\ e^{2\phi} &= \bar{g}_s^2 \frac{(u/R)^2}{1+u^2L^2}, \\ \sum_{a=1}^{3} B_{5\theta_a} d\theta_a &= -\frac{u^2 L}{1+u^2L^2}(a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3), \end{aligned} \tag{4.12}$$

where θ_i's are angular coordinates parameterizing sphere S^3 transverse to the brane, and

$$a_1 = \cos\theta_2 , \qquad a_2 = -\sin\theta_1 \cos\theta_1 \sin\theta_2 , \qquad a_3 = \sin^2\theta_1 \sin^2\theta_2 . \tag{4.13}$$

To study the phase structure of the D5-brane theory, we realize that there are three energy scales which involved. The first one is the energy scale in which the dipole effects become important, that is $u \sim 1/L$. The second one is the energy in which the curvature is of order one $u \sim 1/\sqrt{\bar{g}_s N}$ and finally the dilaton is of order one while the dipole effects are negligible at $u \sim \sqrt{N/\bar{g}_s}$. Using these information we recognize three different phase structures for the theory as following:

1. Suppose $\sqrt{N/\bar{g}_s} \ll 1/L$. At extreme IR regime the theory can be described by perturbative SYM theory. As we increase the energy at the scale $u \sim 1/\sqrt{\bar{g}_s N}$ the

curvature is of order one and then we have to change our description. For $1/\sqrt{\bar{g}_s N} \ll u \ll \sqrt{N/\bar{g}_s}$ the good description is given by gravity solution of D5-brane. For the energy of order $\sqrt{N/\bar{g}_s}$ the dilation is of order one, thus by passing this energy we need to change our gravity description using S-duality. Therefore for $u \gg \sqrt{N/\bar{g}_s}$ NS5-brane description is valid. As we keep increasing the energy the effects of B field become important, and eventually at UV we will have a theory which could be considered as a new deformation of little string theory. We will return to this solution in the next section.

2. When $1/\sqrt{\bar{g}_s N} \ll 1/L \ll \sqrt{N/\bar{g}_s}$. The IR regime is again described by perturbative SYM theory. For $1/\sqrt{\bar{g}_s N} \ll u \ll 1/L$ the curvature and dilation are small. Moreover the effects of B field are still negligible. So, the theory can be described by gravity solution of ordinary D5-brane. As we pass the scale $1/L$ the effects of B field become important. From the gravity solution (4.12) we see that the dilation remains small all the time and therefore the good description is given by gravity solution of D5-brane in the presence of B field (4.12).

3. For the case of $1/L \ll 1/\sqrt{\bar{g}_s N}$, the extreme IR regime is described by perturbative SYM theory. At the scale of $1/L$ the curvature is still large but the dipole effects become important and thus the good description in region $1/L \ll u \ll 1/\sqrt{\bar{g}_s N}$ is given by perturbative dipole gauge theory. By passing the energy scale $1/\sqrt{\bar{g}_s N}$ the curvatures becomes small while the effects of B field are important. Therefore for energy $u \gg 1/\sqrt{\bar{g}_s N}$ the good description is given by gravity solution of D5-brane in the presence of B field (4.12).

A new feature of 6-dimensional dipole field theory is that there is a regime in which we have perturbative dipole gauge theory description. We note that this is not the case when we have noncommutative brane [13]. The reason is that in the noncommutative case the scale in which the effects of noncommutativity are important is written in terms of g_{eff} and it is not possible to have $g_{\text{eff}} \ll 1$ while the noncommutative effects remain important. In other words, in the phase diagram we do not have a range of energy where the perturbative noncommutative gauge theory is valid [13].

4.3 D4-brane

Having the supergravity solution for D5-brane with 8 supercharges, we can easily find a solution for D4-brane which preserves 8 supercharges as well. Here the matrix M representing the dipoles can be written as

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L & 0 & 0 \\ 0 & -L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L \\ 0 & 0 & 0 & -L & 0 \end{pmatrix}. \tag{4.14}$$

Plugging this matrix into the supergravity solution (4.2), one finds

$$
\begin{aligned}
l_s^2 ds^2 &= \left(\frac{u}{R}\right)^{3/2}\left(-dt^2+dx_1^2+\cdots+dx_3^2+\frac{dx_4^2}{1+u^2L_{\rm eff}^2}\right)+\\
&\quad+\left(\frac{R}{u}\right)^{3/2}\left(du^2+u^2d\Omega_3^2-\frac{u^4L_{\rm eff}^2}{1+u^2L_{\rm eff}^2}(a_2d\theta_2+a_3d\theta_3+a_4d\theta_4)^2\right),\\
e^{2\phi} &= \bar{g}_s^2\frac{(u/R)^{3/2}}{1+u^2L_{\rm eff}^2}\,,\\
\sum_{a=2}^{4}B_{4\theta_a}d\theta_a &= -\frac{u^2L_{\rm eff}}{1+u^2L_{\rm eff}^2}(a_2d\theta_2+a_3d\theta_3+a_4d\theta_4)\,,
\end{aligned}
\tag{4.15}
$$

where $\theta_1,\dots,\theta_4$ are angular coordinates parameterizing the sphere S^4 transverse to the brane, and

$$
a_2=\cos\theta_3\,,\qquad a_3=-\sin\theta_2\cos\theta_2\sin\theta_3\,,\qquad a_4=\sin^2\theta_2\sin^2\theta_3\,. \tag{4.16}
$$

Moreover the effective dipole vector is defined by $L_{\rm eff}=L\sin\theta_1$. As we see there are a locus of zeros given by $\sin\theta_1$ in which the effective dipole is zero. We note that the effective dipole vector seen from supergravity is smaller than the naively expected dipole vector L. This can be interpreted as due to strong interactions. This is very similar to what we have in the noncommutative gauge theory case [11].

The three energy scales that involved in parameterizing the phase diagram of the theory are as following. At $u\sim 1/\bar{g}_sN$ the curvature is of order one. The dipole effects become important at scale $u\sim 1/L_{\rm eff}$, while the dipole effects are negligible the dilaton is of order one at $u\sim N^{1/3}/\bar{g}_s$. The same as D5-brane case we distinguish three different phase structures of the theory as following:

1. When $1/L_{\rm eff}\gg N^{1/3}/\bar{g}_s$ the theory at IR limit is described by perturbative 5-dimensional SYM theory. In flow from low energy to high energy we need to change our description from perturbative SYM theory to gravity description at energy $u\sim 1/\bar{g}_sN$ where the curvature becomes small. In fact the good description of the theory in energy range $1/\bar{g}_sN\ll u\ll N^{1/3}/\bar{g}_s$ is given by gravity solution of D4-brane. As we pass energy $u\sim N^{1/3}/\bar{g}_s$ the dilaton gets large and we have to lift the theory to eleven dimensional SUGRA and the uplifted D4-brane solution is a good description. Eventually at the scale $u\sim 1/L_{\rm eff}$ the B field effects become important. Because of these effects the dilaton becomes small at $u\sim \bar{g}_s^3/NL_{\rm eff}^4$ and therefore the good description at UV will be given by the supergravity solution of D4-brane in the presence of B field with one leg along the worldvolume and the other transverse to it.

2. For the case of $1/\bar{g}_sN\ll 1/L_{\rm eff}\ll N^{1/3}/\bar{g}_s$ the IR limit is described by perturbative 5-dimensional SYM theory. As we increase the energy we have to change our description from perturbative SYM theory to the gravity description at scale $u\sim 1N^{1/3}/\bar{g}_s$. At scale $1/L_{\rm eff}$ the B field effects become important while the dilaton remains small all the time. As a result the good description at UV is given by type-IIA supergravity

solution of D4-brane in the presence of B field with one leg along the brane and the other transverse to it.

3. For $1/L_{\rm eff} \ll 1/\bar{g}_s N$ at extreme IR limit the perturbative SYM theory is valid. As we increase the energy at scale $u \sim 1/L_{\rm eff}$ the good description is given by perturbaive dipole gauge theory. This description is valid till we reach the energy $u \sim 1/\bar{g}_s N$ where the curvature is of order one and we have to change our description from perturbative dipole gauge theory to gravity solution of D4-brane in the presence of B field. Since the dilation remains small all the time this is also the valid description at UV limit.

Again, in comparison with noncommutative field theory we see that for dipole field theory there is a range of energy in which the good description is given by perturbative dipole gauge theory.

4.4 D2-brane

One can generalize the previous solutions to the supergravity solution of D2-brane in the presence of B field with one leg along the worldvolume that preserves 8 supercharges. The solution is

$$\begin{aligned}
l_s^2 ds^2 &= \left(\frac{u}{R}\right)^{5/2}\left(-dt^2+dx_1^2+\frac{dx_2^2}{1+u^2L_{\rm eff}^2}\right)+\\
&\quad+\left(\frac{R}{u}\right)^{5/2}\left(du^2+u^2d\Omega_6^2-\frac{u^4L_{\rm eff}^2}{1+u^2L_{\rm eff}^2}(a_{\theta_4}d\theta_4+a_{\theta_5}d\theta_5+a_{\theta_6}d\theta_6)^2\right),\\
e^{2\phi} &= \bar{g}_s^2\frac{(R/u)^{5/2}}{1+u^2L_{\rm eff}^2},\\
\sum_{a=4}^{6}B_{2\theta_a}d\theta_a &= -\frac{u^2L_{\rm eff}}{1+u^2L_{\rm eff}^2}(a_4d\theta_4+a_5d\theta_5+a_6d\theta_6)\,,
\end{aligned}\tag{4.17}$$

where $\theta_1,\ldots,\theta_6$ are angular coordinates parameterizing the sphere S^6 transverse to the brane, and

$$a_4=\cos\theta_5\,,\qquad a_5=-\sin\theta_4\cos\theta_4\sin\theta_5\,,\qquad a_6=\sin^2\theta_4\sin^2\theta_5\,. \tag{4.18}$$

The effective dipole vector is defined by $L_{\rm eff}=L\sin\theta_1\sin\theta_2\sin\theta_3$. The dipole vector could be zero as locus where $\sin\theta_1\sin\theta_2\sin\theta_3$ vanishes. In this locus the gravity solution is locally the same as ordinary D-brane solution [24].

The three scales which play role in studying of the phase structure of D2-brane are as following. The curvature is of order one at $u \sim \bar{g}_s N$. The dipole effects are negligible below the energy of order $u \sim 1/L_{\rm eff}$ and for the case where the dipole effects are negligible the dilaton is of order one at $u \sim \bar{g}_s/N^{1/5}$. Using these information, one can proceed to analyze the phase structure of the theory.

1. When $1/L_{\rm eff} \gg \bar{g}_s N$ the good description at UV is given by perturbative 3 dimensional dipole gauge theory with 8 supercharges. In flow from high energy to low

energy the dipole effects become negligible at $u \sim 1/L_{\rm eff}$ and therefore at energy range $\bar{g}_s N \ll u \ll 1/L_{\rm eff}$ the good description is given by perturbative SYM theory where we have supersymmetry enhancement. At energy scale $u \sim \bar{g}_s N$ the curvature is of order one and we need to change our description from perturbative SYM theory to gravity solution of D2-brane. The gravity solution of D2-brane is a good description till we reach scale $u \sim \bar{g}_s/N^{1/5}$ where the dilaton grows and we need to lift the theory to eleven dimensional SUGRA and thus the good description is given by uplifted D2-brane. Finally at IR regime the theory flows to a fixed point which is given by $\mathcal{N}=2$ SCFT in three dimensions.

2. When $\bar{g}_s/N^{1/5} \ll 1/L_{\rm eff} \ll \bar{g}_s N$ the theory is described by perturbative 3-dimensional supersymmetric dipole gauge theory for energy above $\bar{g}_s N$. At this energy the curvature is of order one and therefore we have to change our description at this scale. In fact the good description of theory is given in terms of supergravity solution of D2-brane in the presence of B field in energy range $1/L_{\rm eff} \ll u \ll \bar{g}_s N$ while below the scale $1/L_{\rm eff}$ the B field effects are negligible and the theory is described by D2-branes solution. At energy $u \sim \bar{g}_s/N^{1/5}$ the dilaton is of order one and thus below this energy the uplifted D2-brane solution is a good description. Eventually the theory reach its fixed point at IR where the good description is given by $\mathcal{N}=2$ SCFT in three dimensions.

3. In the case of $1/L_{\rm eff} \ll \bar{g}_s/N^{1/5}$ the UV regime is described by perturbative supersymmetric dipole gauge theory. As we decrees the energy the effective dimensionless gauge coupling grows at $\sim \bar{g}_s N$ where we have to change our description to the gravity description. The theory is described by supergravity solution of D2-brane in the presence of B field until we reach the energy scale $u \sim \bar{g}_s^{5/9} N^{1/9}/L_{\rm eff}^{4/9}$ where the dilaton start growing and the good description will be given by uplifted D2-brane in the presence of C field. The dipole effects become negligible at $1/L_{\rm eff}$ and finally the IR limit is described by the fixed point of the theory which is $\mathcal{N}=2$ SCFT in three dimensions.

4.5 D1-brane

Using the same method as before the supergravity solution of D1-brane in the presence of a B field with one leg along brane and the other transverse to it which preserves 8 supercharges can be read from (4.2), that is

$$\begin{aligned}
l_s^2 ds^2 &= \left(\frac{u}{R}\right)^3 \left(-dt^2 + \frac{dx_1^2}{1+u^2L_{\rm eff}^2}\right) + \\
&\quad + \left(\frac{R}{u}\right)^3 \left(du^2 + u^2 d\Omega_7^2 - \frac{u^4 L_{\rm eff}^2}{1+u^2L_{\rm eff}^2}(a_5 d\theta_5 + a_6 d\theta_6 + a_7 d\theta_7)^2\right), \\
e^{2\phi} &= \bar{g}_s^2 \frac{(R/u)^6}{1+u^2L_{\rm eff}^2}\,, \\
\sum_{a=5}^{7} B_{1\theta_a} d\theta_a &= -\frac{u^2 L_{\rm eff}}{1+u^2L_{\rm eff}^2}(a_5 d\theta_5 + a_6 d\theta_6 + a_7 d\theta_7)\,,
\end{aligned} \tag{4.19}$$

where $\theta_1, \ldots, \theta_7$ are angular coordinates parameterizing the sphere S^7 transverse to the brane, and

$$a_5 = \cos\theta_6\,, \qquad a_6 = -\sin\theta_5\cos\theta_5\sin\theta_6\,, \qquad a_7 = \sin^2\theta_5\sin^2\theta_6\,. \tag{4.20}$$

The effective dipole vector is defined by $L_{\rm eff} = L\sin\theta_1\sin\theta_2\sin\theta_3\sin\theta_4$. The dipole vector could be zero in locus where $\sin\theta_1\sin\theta_2\sin\theta_3\sin\theta_4$ vanishes. In this locus the gravity solution is locally the same as ordinary D-brane solution [24].

As it can be seen from the supergravity solution (4.19) the curvature is of order one at $u \sim g_{\rm YM}\sqrt{N}$. The dipole effects are important at the scale $u \sim 1/L_{\rm eff}$. While the dipole effects are negligible the dilaton is of one at $u \sim g_{\rm YM}N^{1/6}$. These are the scales that involved in phase structure of D1-brane. By making use of these information we can work out the phase structure of the theory as following.

1. For the case of $1/L_{\rm eff} \gg g_{\rm YM}\sqrt{N}$ the good description at UV is given by perturbative (1+1)-dimensional supersymmetric dipole gauge theory. As we flow from high energy to low energy the dipole effects become negligible at $u \sim 1/L_{\rm eff}$. In fact for $g_{\rm YM}\sqrt{N} \ll u \ll 1/L_{\rm eff}$ the SYM theory in (1+1)-dimensional can be trusted. As we approach the low energy the type-IIB supergravity solution (4.19) can be trusted in the region $g_{\rm YM}N^{1/6} \ll u \ll g_{\rm YM}\sqrt{N}$. In the region $u \ll g_{\rm YM}N^{1/6}$ the dilaton is large and we need the S-dual picture. Using S-duality the solution (4.19) maps to the fundamental string solution. In IR limit the string coupling vanishes while the curvature in the new string unit behaves as $l_s^2\mathcal{R} \sim g_{\rm YM}^2/u^2$. This means that the supergravity description of fundamental string breaks down for small u and in fact the IR limit is a trivial orbifold (R^{8N}/S_N) conformal field theory [24].

2. When $g_{\rm YM}N^{1/6} \ll 1/L_{\rm eff} \ll g_{\rm YM}\sqrt{N}$ the perturbative (1+1)-dimensional dipole gauge theory is valid for $u \gg g_{\rm YM}\sqrt{N}$. While for for $1/L_{\rm eff} \ll u \ll g_{\rm YM}\sqrt{N}$ the curvature and dilaton are small and the supergarvity solution of D1-brane with B field is a good description. For $g_{\rm YM}N^{1/6} \ll u \ll 1/L_{\rm eff}$ the gravity is still a good description but the B field effects are negligible. For the energy $u \ll g_{\rm YM}N^{1/6}$ the situation is the same as previous case.

3. When $1/L_{\rm eff} \ll g_{\rm YM}N^{1/6}$ we can trust perturbative (1+1)-dimensional dipole gauge theory for $u \gg g_{\rm YM}\sqrt{N}$. In the region of $(g_{\rm YM}^6N)^{1/8}/L_{\rm eff}^{1/4} \ll u \ll_{\rm YM} \sqrt{N}$ the curvature is small and dilaton is small too and the good description is given by supergravity solution (4.19). For the energy of $u \ll g_{\rm YM}^6N)^{1/8}/L_{\rm eff}^{1/4}$ the dilaton is large while the B field effects are important and therefore we need to use the S-dual picture which maps the solution (4.19) to the supergravity solution of fundamental string in the presence of RR 2-form with one leg along the string and the other transverse to it. The B field effects becomes negligible for $u \ll 1/L_{\rm eff}$ where the gravity solution of fundamental string is valid. Eventually for $u \ll g_{\rm YM}$ the supergravity description of fundamental string breaks down and in fact the IR limit is a trivial orbifold (R^{8N}/S_N) conformal field theory.

5. Fivebranes

In this section we shall discuss possible dipole deformation of NS5-branes and M5-branes worldvolume theories.

5.1 NS5-brane

Now we want to study the theory on the type-II NS5-branes in the presence of different RR fields with one leg along the transverse direction to the NS5-branes and the others along the worldvolume. The supergravity solution of NS5-branes in the presence of RR $(6-p)$-form, for $p = 0, \ldots, 4$, with one leg along the transverse directions and $(5-p)$ legs along the NS5-branes worldvolume is given by

$$\begin{aligned} ds^2 &= (1+u^2 n^T M^T M n)^{1/2} \Bigg[\sum_{i=0}^{p} dx_i^2 + \frac{\sum_{j=p+1}^5 dx_j^2}{1+u^2 n^T M^T M n} + \\ &\qquad + g_s^2 l_s^4 f \left(du^2 + u^2 dn^T dn - \frac{u^4 (n^T M^T dn)^2}{1+u^2 n^T M^T M n} \right) \Bigg] , \\ e^{2\phi} &= g_s^2 (1+u^2 n^T M^T M n)^{(p-2)/2} f , \\ \sum_{a=6}^{9} C_{(p+1)\cdots 5a} dx_a &= - \frac{u^2 dn^T M n}{1+u^2 n^T M^T M n} , \end{aligned} \tag{5.1}$$

where $dx_0^2 = -dt^2$, $C_{(p+1)\cdots 5a}$ is $(6-p)$-form and $f = 1 + c_5 N / g_s^2 l_s^2 u^2$. The energy coordinat u is related to the radial coordinat r by $u = r/g_s l_s^2$. A way to find these solutions is to start with type-IIB D5-branes in the presence of B field with one leg along the worldvolume. Then using S-duality we can find the supergravity solution of type-IIB NS5-branes in the presence of RR 2-form with one leg along the worldvolume. Other solutions can be obtained by T-duality. Under S-duality we have

$$l_s^2 \to g_s l_s^2 , \qquad g_s \to \frac{1}{g_s} . \tag{5.2}$$

Therefore the decoupling limit of above supergravity solutions can be defined as the limit $g_s \to 0$, keeping the following quantities fixed

$$u = \frac{r}{g_s l_s^2} , \qquad l_s = \text{fixed} . \tag{5.3}$$

In this limit, setting M as (4.11), the above supergravity solution becomes

$$\begin{aligned} ds^2 &= (1+u^2 L^2)^{1/2} \Bigg[\sum_{i=0}^{p} dx_i^2 + \frac{\sum_{j=p+1}^5 dx_j^2}{1+u^2 L^2} + \\ &\qquad + \frac{N l_s^2}{u^2} \Bigg(du^2 + u^2 d\Omega_3 - \\ &\qquad\qquad - \frac{u^4 L^2}{1+u^2 L^2} (a_1 d\theta_1 + a_3 d\theta_1 + a_3 d\theta_3)^2 \Bigg) \Bigg] , \end{aligned}$$

$$
\begin{aligned}
e^{2\phi} &= \frac{N}{l_s^2 u^2}(1+u^2L^2)^{(p-2)/2}\,, \\
\sum_{a=6}^{9} C_{(p+1)\cdots 5\theta_a} d\theta_a &= -\frac{u^2 L}{1+u^2L^2}(a_1 d\theta_1 + a_3 d\theta_1 + a_3 d\theta_3)\,,
\end{aligned}
\tag{5.4}
$$

where a_i's are defined the same as (4.13). This solution can be thought as a new deformation of little string theory. The curvature of the metric is given by

$$
l_s^2 \mathcal{R} \sim \frac{1}{N}\frac{1}{(1+u^2L^2)^{1/2}}\,. \tag{5.5}
$$

Therefore for large N the curvature is small and one can trust the supergravity description. For $uL \gg 1$ the gravity approximation can be trusted for finite N.

As an application let us to calculate the absorption cross section of polarized gravitons. In general one can show that in these backgrounds the scattering potential for a graviton polarized along the brane directions is

$$
V(\rho) = -1 + \left(\frac{3}{4} - N\omega^2 l_s^2\right)\frac{1}{\rho^2}\,, \tag{5.6}
$$

where $\rho = r\omega$ and ω is the energy of incoming waves. Therefore we see that after the decoupling limit the absorption cross section can be nonzero only for waves with energy ω^2 larger than $\sim 1/Nl_s^2$. Essentially the same effect appears in the little string theory and one can see that the theory has a mass gap of order $M_{\rm gap}^2 \sim 1/Nl_s^2$ [31] which is exactly the same as little string theory. In other words, the mass gap of the theory is independent of the dipole vector. The mass gap in deformed little string theory has been also studied in [32, 33].

5.2 M5-branes

To find an eleven dimensional supergravity solution corresponding to the M5-branes in the presence of a C field with two legs along the worldvolume and one leg transverse to it, we can start from D_4 brane solution and then lifting it to 11-dimensional supergravity and sending the radius of $11th$ direction to infinity, $R_{11} \to \infty$. In this limit setting $R_{11}M = \bar{M}$, one finds:

$$
\begin{aligned}
ds_{11}^2 &= \left(1+\frac{r^2}{l_p^6} n^T \bar{M}^T \bar{M} n\right)^{1/3}\Bigg[f^{-1/3}\left(-dt^2+\cdots+dx_3^2+\frac{dx_4^2+dx_5^2}{1+\frac{r^2}{l_p^6}n^T\bar{M}^T\bar{M}n}\right)+ \\
&\qquad + f^{2/3}\left(dr^2 + r^2 dn^T dn - \frac{\frac{r^4}{l_p^6}(n^T\bar{M}dn)^2}{1+\frac{r^2}{l_p^6}n^T\bar{M}^T\bar{M}n}\right)\Bigg]\,, \\
\sum_{a=2}^{4} C_{45a}dx^a &\sim -\frac{\frac{r^2}{l_p^6}dn^T\bar{M}n}{1+\frac{r^2}{l_p^6}n^T\bar{M}^T\bar{M}n}\,,
\end{aligned}
\tag{5.7}
$$

where

$$
f = 1 + \frac{\pi N l_p^3}{r^3}\,. \tag{5.8}
$$

The decoupling limit of the theory is defined as a limit where $l_p \to 0$ keeping $u = r/l_p^3$ fixed. In this limit, setting $\bar{M}$ as

$$\bar{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{L} & 0 & 0 \\ 0 & -\bar{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{L} \\ 0 & 0 & 0 & -\bar{L} & 0 \end{pmatrix}, \tag{5.9}$$

the above supergravity solution reads

$$\begin{aligned} l_s^2 ds^2 &= (1+u^2 L_{\text{eff}}^2)^{1/2} \Bigg[\frac{u}{(\pi N)^{1/3}} \left(-dt^2 + dx_1^2 + \cdots + dx_3^2 + \frac{dx_4^2 + dx_5^2}{1+u^2 L_{\text{eff}}^2} \right) + \\ &\qquad + \frac{(\pi N)^{2/3}}{u^2} \Bigg(du^2 + u^2 d\Omega_3^2 - \\ &\qquad\qquad - \frac{u^4 L_{\text{eff}}^2}{1+u^2 L_{\text{eff}}^2} (a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4)^2 \Bigg) \Bigg], \\ \sum_{a=2}^{4} C_{45\theta_a} d\theta_a &= -\frac{u^2 L_{\text{eff}}}{1+u^2 L_{\text{eff}}^2} (a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4), \end{aligned} \tag{5.10}$$

where $\theta_1, \ldots, \theta_4$ are angular coordinates parameterizing the sphere S^4 transverse to the brane and a_i's are given by (4.16). The effective "*discpole*" is also defined by $L_{\text{eff}} = \bar{L} \sin\theta_1$ where L has dimension of $(\text{length})^2$.

The curvature of the metric in (5.10) is

$$l_p^2 \mathcal{R} \sim \frac{1}{N^{2/3}} \frac{1}{(1+u^2 L_{\text{eff}}^2)^{1/3}}. \tag{5.11}$$

Thus we can trust the supergravity solution for large N. This solution can provide a dual description of a deformation of $(2,0)$ theory. The dipole deformation of $(2,0)$ has also been considered in [22] as "*discpole theory*".

6. Wilson loop

In this section we use dual gravity description of dipole gauge theory to compute Wilson loop for different brane theories. According to the AdS/CFT correspondence the Wilson loop of the gauge theory can be computed in dual string theory description by evaluating the partition function of string whose worldsheet is bounded by the loop [34, 35]. In the supergravity approximation the dominant contribution comes from the minimal two dimensional surface bounded by the loop. The expectation value of Wilson loop is

$$\langle W(C) \rangle \sim e^{-S}, \tag{6.1}$$

where S is string action evaluated on the minimal surface bounded by loop C.

Now we would like to compute the Wilson loop in the dipole gauge theories living on the worldvolume of Dp-brane using supergravity solution (4.2). The string action is given by

$$S = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \sqrt{-\det\left(G_{\mu\nu}\partial_i X^\mu \partial_j X^\nu\right)}\,. \tag{6.2}$$

We parametrized the string configuration by $\tau = t, \sigma = u$ and $x_p = x(u)$. In this parameterization, using the supergravity solution (4.2), the string action (6.2) reads

$$S = \frac{1}{2\pi} \int dt du \sqrt{1 + \frac{(u/R)^{7-p}}{1+u^2 L_{\text{eff}}^2}(\partial_u x)^2}\,, \tag{6.3}$$

where $L_{\text{eff}}^2 = n^T M^T M n$ is effective dipole vector. First of all from this equation we find that despite the theory is non-local, the end points of string can be fixed at large u. We note that in the noncommutative gauge theory where we have a non-zero B field with both legs along the brane worldvolume we have problem for fixing the end points [11], though we could fix it using moving frame [13].

The action (6.3) is minimized when

$$\frac{(u/R)^{7-p}}{1+u^2 L_{\text{eff}}^2} \frac{\partial_u x}{\left(1 + \frac{(u/R)^{7-p}}{1+u^2 L_{\text{eff}}^2}(\partial_u x)^2\right)^{1/2}} = \text{constant} = \left(\frac{(u_0/R)^{7-p}}{1+u_0^2 L_{\text{eff}}^2}\right)^{1/2}, \tag{6.4}$$

where u_0 is the point where $\partial_u x|_{u_0} \to \infty$. This equation can be solved for $\partial_u x$, that is

$$\partial_u x = \frac{\left(\frac{1+u^2 L_{\text{eff}}^2}{(u/R)^{7-p}}\right)^{1/2}}{\sqrt{\left(\frac{u}{u_0}\right)^{7-p} \frac{1+u_0^2 L_{\text{eff}}^2}{1+u^2 L_{\text{eff}}^2} - 1}}\,. \tag{6.5}$$

Hence

$$x(u) = \int_{u_0}^{u} du \frac{\left(\frac{1+u^2 L_{\text{eff}}^2}{(u/R)^{7-p}}\right)^{1/2}}{\sqrt{\left(\frac{u}{u_0}\right)^{7-p} \frac{1+u_0^2 L_{\text{eff}}^2}{1+u^2 L_{\text{eff}}^2} - 1}}\,. \tag{6.6}$$

The $Q\bar{Q}$ separation is defined by

$$\frac{l}{2} := x(u \to \infty) = \frac{R^{(7-p)/2}}{u_0^{(5-p)/2}} \int_1^\infty dy \frac{(1+y^2 \bar{L}_{\text{eff}}^2)^{1/2}}{y^{(7-p)/2}\sqrt{y^{7-p}\frac{1+\bar{L}_{\text{eff}}^2}{1+y^2 \bar{L}_{\text{eff}}^2} - 1}}\,. \tag{6.7}$$

Here $\bar{L}_{\text{eff}} = u_0 L_{\text{eff}}$. Using (6.3) we can calculate the energy of the $Q\bar{Q}$ system as following

$$E = \frac{u_0}{\pi}\left[\int_1^\infty dy \left(\frac{\left(y^{7-p}\frac{1+\bar{L}_{\text{eff}}^2}{1+y^2 \bar{L}_{\text{eff}}^2}\right)^{1/2}}{\sqrt{y^{7-p}\frac{1+\bar{L}_{\text{eff}}^2}{1+y^2 \bar{L}_{\text{eff}}^2} - 1}} - 1\right) - 1\right]. \tag{6.8}$$

Here we subtracted the infinity coming from mass of the W-boson which corresponds to string stretching all the way to $u = \infty$.

When the distance between quark and anti-quark is much bigger than their dipole size, the energy is given by

$$E \sim -\left(\frac{g_{\rm YM}^2 N}{l^2}\right)^{1/(5-p)} \left(1 + c_0 L_{\rm eff}^2 \left(\frac{g_{\rm YM}^2 N}{l^2}\right)^{2/(5-p)} + \cdots\right), \tag{6.9}$$

where c_0 is a numerical constant. The first term in the above expression is what we have in the ordinary gauge theory [36] and the second term can be interpreted as the dipole-dipole interaction.[1]

7. Conclusion

In this paper we have studied the Dp-branes supergravity solution in the presence of non-zero B field with one one leg along the worldvolume and the other transverse to it. We have shown that for $p \leq 5$ the theory one the brane decouples form the gravity, therefore the supergravity solution provides a dual description of worldvolume theory which is noncommutative dipole gauge theory. This supergravity solution, in general, breaks the supersymmetry. Nevertheless one can consider the case in which we left with some supersymmetries. In this paper we only considered those brane solutions which preserve 8 supercharges. Having a general gravity solution it is not clear if the solution is stable. Nevertheless since we are considering those brane systems which are supersymmetric the solutions we found are stable.

We also studied the type-II NS5-branes in the presence of non-zero RR field with one leg along the transverse direction to the NS5-branes and the others along their worldvolume. These solutions can be thought as a deformation of little string field theory. These theories also exhibit the same mass gap as little string theory.

We have also studied the M5-brane in the presence of a C field with two legs along the worldvolume and one leg along the transverse directions. Up on compactification on a circle we will find the 10-dimensional supergravity solution studied in this paper. We note, however, that there is an other solution which we have not considered in this paper. This solution can be found by compactifying the M5-brane solution (5.10) on a worldvolume direction in which the C field is not defined. In the type-IIA limit this corresponds to a D4-brane solution in the presence of RR 3-form with two legs along the worldvolume and one leg transverse to it. There are also other interesting solutions both in type-II string theories and M-theory where one leg of B field or C field is along the time direction. We will back to these questions in our incoming paper [38].

We have also computed the Wilson loop of the dipole gauge theory using dual gravity description. We saw that despite the fact that the theory is non-local we can fix the end points of strings at large u. Note that this is not the case for noncommutative field theory. Therefore we might conclude that the dipole gauge theory is a non-local deformation of ordinary gauge theory which could be much more simpler than the non-local deformation obtained by making the space to be noncommutative, i.e. noncommutative gauge theory.

[1]The Wilson loop of the dipole gauge theory has been also considered by M.M. Sheikh-Jabbari [37].

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